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Two-Form Gravity and the Generation of Space-Time

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ABSTRACT

In the framework of the two-form gravity, which is classically equivalent to the Einstein gravity, the one-loop effective potential for the conformal factor of metric is calculated in the finite volume and in the finite temperature by choosing a temporal gauge condition. There appears a quartically divergent term which cannot be removed by the renormalization of the cosmological term and we find there is only one non-trivial minimum in the effective potential. If the cut-off scale has a physical meaning, *e.g.* the Planck scale coming from string theory, this minimum might explain why the space-time is generated, *i.e.* why the classical metric has a non-trivial value.

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1 Introduction

Although the classical theory of the Einstein gravity is simple and successfully describes the nature, we encounter the many serious problems when we try to construct a quantum theory of gravity. One of these problems is that the action of the Einstein gravity is not renormalizable. This might suggest that the Einstein gravity theory is an effective theory obtained from a more fundamental theory, *e.g.* string theory. Two-form gravity theory is known to be classically equivalent to the Einstein gravity theory and is obtained from a topological field theory, which is called BF theory [1], by imposing constraint conditions [2]. The characteristic feature of the BF theory is that the system has the Kalb-Ramond symmetry [3], which is a large local symmetry. The Kalb-Ramond symmetry can be considered to reflect the stringy structure of the fundamental gravity theory [4].

In this paper, we calculate, in the framework of the two-form gravity, the one-loop effective potential for the conformal factor of the metric in the finite volume and in the finite temperature by choosing a temporal gauge fixing condition. In the effective potential, there appears a quartically divergent term which cannot be removed by the renormalization of the cosmological term. We also find that there is only one non-trivial minimum in the effective potential, which might explain why the space-time is generated, *i.e.* why the classical metric has a non-trivial value if the cut-off scale has a physical meaning, *e.g.* the Planck scale coming from string theory.

In the next section, we explain the action of the two-form gravity and clarify the gauge symmetries of the system. In section 3, we fix the gauge symmetries and expand the action around a classical solution. The measures which keeps the gauge symmetries and the action of the ghost fields are given in section 4. In section 5, the one-loop effective potential for the conformal factor of metric is calculated. The last section is devoted to the summary.

2 Two-Form Gravity Action

We start with the following action which describes a topological field theory called BF theory [1]:

$$S = \int d^4x \frac{1}{2}\epsilon^{\mu\nu\lambda\rho} \left(B^a_{\mu\nu}(x) R^a_{\lambda\rho}(x) + \bar{B}^a_{\mu\nu}(x) \bar{R}^a_{\lambda\rho}(x) \right) . \tag{1}$$

Here $B^a_{\mu\nu}(x)$ and $\bar{B}^a_{\mu\nu}(x)$ are two-form fields and $R^a_{\mu\nu}(x)$ and $\bar{R}^a_{\mu\nu}(x)$ are $SU(2)$ field strength (a denotes an $SU(2)$ index $a = 1, 2, 3$) given by the $SU(2)$ gauge fields A and $\bar{A}$:

$$R^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g\epsilon^{abc} A^b_\mu A^c_\nu \ , \qquad \bar{R}^a_{\mu\nu} = \partial_\mu \bar{A}_\nu - \partial_\nu \bar{A}_\mu + g\epsilon^{abc} \bar{A}^b_\mu \bar{A}^c_\nu \ . \tag{2}$$

Here g is a gauge coupling constant. The gauge fields A and $\bar{A}$ are identified with spin connections. The action (1) has $SU(2) \times SU(2)$ gauge symmetry corresponding to local Lorentz symmetry $SO(4) \sim SU(2) \times SU(2)$. Besides $SU(2) \times SU(2)$ gauge transformation, the action is invariant under the Kalb-Ramond symmetry transformation [3]:

$$\left\{ \begin{array}{ccc} A^a_\mu & \to & A^a_\mu \\ B^a_{\mu\nu} & \to & B^a_{\mu\nu} + \nabla^{ab}_\mu \Lambda^b_\nu - \nabla^{ab}_\nu \Lambda^b_\mu \ , \end{array} \right. \tag{3}$$

$$\left\{ \begin{array}{ccc} A^a_\mu & \to & A^a_\mu \\ B^a_{\mu\nu} & \to & B^a_{\mu\nu} + \nabla^{ab}_\mu \Lambda^b_\nu - \nabla^{ab}_\nu \Lambda^b_\mu \ . \end{array} \right. \tag{4}$$

Here the covariant derivative ∇^{ab}_μ is defined by $\nabla^{ab}_\mu \Lambda^b_\nu = \partial_\mu \Lambda^a_\nu + \epsilon^{abc} A^b_\mu \Lambda^c_\nu$.

The action (1) is known to be equivalent to the Einstein action if we impose the following constraints [2]:

$$\epsilon^{\mu\nu\lambda\rho}(B^a_{\mu\nu} B^b_{\lambda\rho} - \frac{1}{3}\delta^{ab} B^c_{\mu\nu} B^c_{\lambda\rho}) = 0 \ , \tag{5}$$

$$\epsilon^{\mu\nu\lambda\rho}(\bar{B}^a_{\mu\nu} \bar{B}^b_{\lambda\rho} - \frac{1}{3}\delta^{ab} \bar{B}^c_{\mu\nu} \bar{B}^c_{\lambda\rho}) = 0 \ , \tag{6}$$

$$\epsilon^{\mu\nu\lambda\rho} B^a_{\mu\nu} \bar{B}^b_{\lambda\rho} = 0 \ . \tag{7}$$

In the following, we only consider the chiral part of the theory, which is given by $B^a_{\mu\nu}$ and A^a_μ, since the anti-chiral part given by $\bar{B}^a_{\mu\nu}$ and $\bar{A}^a_\mu$ is a copy of chiral part and the two parts are decoupled with each other.

We impose the constraint (5) by a multiplier field ϕ^{ab}:

$$S = \int d^4x \frac{1}{2} \epsilon^{\mu\nu\lambda\rho} \left(B^a_{\mu\nu}(x) R^a_{\lambda\rho}(x) + \phi^{ab}(x)(B^a_{\mu\nu} B^b_{\lambda\rho} - \frac{1}{3}\delta^{ab} B^c_{\mu\nu} B^c_{\lambda\rho}) \right) \ . \tag{8}$$

The action (8) is not invariant under the Kalb-Ramond transformation (3) but we can keep the symmetry by introducing a new field G^a_μ, which is transformed by

$$\delta G^a_\mu = \Lambda^a_\mu \tag{9}$$

and by modifying the action:

$$
\begin{aligned}
S &= \int d^4x \epsilon^{\mu\nu\rho\sigma}\Big[B^a_{\mu\nu}R^a_{\rho\sigma} \\
&\quad +\phi^{ab}\{(B^a_{\mu\nu}-\nabla_\mu G^a_\nu+\nabla_\nu G^a_\mu)(B^b_{\rho\sigma}-\nabla_\rho G^b_\sigma+\nabla_\sigma G^b_\rho) \\
&\quad -\frac{1}{3}\delta^{ab}(B^c_{\mu\nu}-\nabla_\mu G^c_\nu+\nabla_\nu G^c_\mu)(B^c_{\rho\sigma}-\nabla_\rho G^c_\sigma+\nabla_\sigma G^c_\rho)\}\Big] \ .
\end{aligned} \tag{10}
$$

We can also consider the cosmological term

$$
S_{\rm cosmo}=\int d^4x\, \Lambda\epsilon^{\mu\nu\rho\sigma}(B^a_{\mu\nu}-\nabla_\mu G^a_\nu+\nabla_\nu G^a_\mu)(B^a_{\rho\sigma}-\nabla_\rho G^a_\sigma+\nabla_\sigma G^a_\rho)\ . \tag{11}
$$

3 Gauge Fixing

The actions (10) and (11) have the following local symmetries: 1. the Kalb-Ramond symmetry, 2. $SU(2)$ gauge symmetry, which is the chiral part of the local Lorentz symmetry, 3. general covariance. The condition

$$
G^a_\mu = 0 \tag{12}
$$

fixes the Kalb-Ramond symmetry. This gauge condition (12) does not generate any ghost action. We also fix the $SU(2)$ gauge symmetry by choosing the temporal gauge condition:

$$
A^a_4 = 0\ . \tag{13}
$$

In the usual gauge theory, the ghost corresponding to the temporal gauge (13) does not contribute to the physical amplitude since the corresponding Jacobian is a c-number. In the two-form gravity theory, however, the Jacobian depends on the space-time metric, as we will see later, and there are contributions from the ghost fields to the physical amplitude.

When the cosmological constant Λ vanishes $\Lambda=0$, a solution of the equations of motion derived from the action (10) is given by

$$
B^a_{\mu\nu}=\varphi^2\eta^a_{\mu\nu}\ , \qquad \text{other fields} = 0 \tag{14}
$$

Here φ is a constant, which cannot be determined from the equations of motion, and $\eta^a_{\mu\nu}$ is the 't Hooft symbol [5] which is defined by,

$$
\eta^a_{\mu\nu} = \begin{cases} \epsilon^{a\mu\nu} & \mu,\nu=1,2,3 \\ \delta^a_\mu & \nu=4 \\ -\delta^a_\nu & \mu=4 \end{cases} \tag{15}
$$

In the following, we calculate the radiative correction [6] of the effective potential for φ by expanding the action around the solution (14) and keeping the quadratic terms:

$$\begin{aligned} S &\sim \int d^4x[2g\varphi^2(A_a^aA_b^b - A_b^aA_a^b) - 2\epsilon^{ijk}b_{ij}^a\partial_4A_k^a + 4\epsilon^{ijk}b_{i4}^a\partial_jA_k^a \\ &\quad +\varphi^2\phi^{ab}\{2b_{ij}^a\epsilon^{ijb} + 2b_{ij}^b\epsilon^{ija} + 4b_{b4}^a + 4b_{a4}^b \\ &\quad -\frac{1}{3}\delta^{ab}(4b_{ij}^c\epsilon^{ijc} + b_{c4}^c)\}] \ . \end{aligned} \tag{16}$$

Here $b_{\mu\nu}^a$ is defined by $B_{\mu\nu}^a = \varphi^2\eta_{\mu\nu}^a + b_{\mu\nu}^a$. The action (16) is invariant under a global $SU(2)$ symmetry. The global $SU(2)$ symmetry is the diagonal part of the $SU(2)$ symmetry coming from the $SU(2)$ gauge symmetry and the $SU(2) \sim SO(3)$ symmetry of the rotation in the three dimensional space. We can now decompose $b_{\mu\nu}^a$ and A_b^a into the diagonal $SU(2)$ spin 0, 1 and 2 components:

$$\left\{ \begin{array}{ccc} 2b_{ij}^a\epsilon^{ijb} & = & \delta^{ab}h + \epsilon^{abc}h^c + h^{ab} \\ 4b_{i4}^a & = & \delta^{ai}e + \epsilon^{aic}e^c + e^{ai} \\ A_b^a & = & \delta^{ab}a + \epsilon^{abc}a^c + a^{ab} \end{array} \right. . \tag{17}$$

By partial integration and the redefinition of the fields, the action (16) is rewritten by,

$$\begin{aligned} S &\sim \int d^4x[12g\varphi^2\tilde{a}^2 + 4g\varphi^2\tilde{a}^a\tilde{a}^a - 2g\varphi^2\tilde{a}^{ab}\tilde{a}^{ab} \\ &\quad -\frac{3}{16g\varphi^2}(\partial_4\tilde{h})^2 - \frac{1}{4g\varphi^2}(\partial_4\tilde{h}^a)^2 + \frac{1}{8g\varphi^2}(\partial_4\tilde{h}^{ab})^2 \\ &\quad -8g\varphi^6(\partial_4^{-1}\tilde{\phi}^{ab})^2 \\ &\quad +\frac{1}{8g\varphi^2}\{(\ \frac{1}{2}\partial_ae^b + \partial_be^a - \frac{2}{3}\delta^{ab}\partial_ce^c) \\ &\quad +\frac{1}{2}(\epsilon^{dcb}\partial_ce^{ad} + \epsilon^{dca}\partial_ce^{bd}) - \partial_4e^{ab}\ \}^2 \ . \end{aligned} \tag{18}$$

The redefined fields are given by

$$\begin{aligned} \tilde{a} &= a + \frac{1}{8g\varphi^2}\partial_4h - \frac{1}{12g\varphi^2}\partial_ae^a \\ \tilde{a}^a &= a^a + \frac{1}{4g\varphi^2}\partial_4a^a + \frac{1}{4g\varphi^2}\partial_ae + \frac{1}{8g\varphi^2}\epsilon^{abc}\partial_be^c - \frac{1}{16g\varphi^2}(\partial_ce^{ca} + \partial_ce^{ac}) \end{aligned}$$

$$
\begin{aligned}
\tilde{a}^{ab} &= a^{ab} - \frac{1}{4g\varphi^2}\partial_4 h^{ab} - \frac{1}{4g\varphi^2}\Big(\partial_a e^b + \partial_b e^a - \frac{2}{3}\delta^{ab}\partial_c e^c\Big) \\
&\quad + \frac{1}{4g\varphi^2}(\epsilon^{dcb}\partial_c e^{ad} + \epsilon^{dca}\partial_c e^{bd}) \\
\tilde{h} &= h - \frac{2}{3}\partial_4^{-1}\partial_a e^a \\
\tilde{h}^a &= h^a + \partial_4^{-1}\Big\{\partial_a e + \frac{1}{2}\epsilon^{abc}\partial_b e^c - \frac{1}{4}(\partial_c e^{ca} + \partial_c e^{ac})\Big\} \\
\tilde{h}^{ab} &= h^{ab} - \partial_4^{-1}\Big\{\frac{1}{2}\Big[\partial_a e^b + \partial_b e^a - \frac{2}{3}\delta^{ab}\partial_c e^c - (\epsilon^{dcb}\partial_c e^{ad} + \epsilon^{dca}\partial_c e^{bd})\Big] \\
&\quad - 8g\varphi^4\partial_4^{-1}\phi^{ab}\Big\} \\
\tilde{\phi}^{ab} &= \phi^{ab} - \partial_4\Big\{\frac{1}{16g\varphi^4}\Big(\partial_a e^b + \partial_b e^a - \frac{2}{3}\delta^{ab}\partial_c e^c - \epsilon^{dcb}\partial_c e^{ad} - \epsilon^{dca}\partial_c e^{bd}\Big) \\
&\quad + \frac{1}{8g\varphi^4}\partial_4 e^{ab}\Big\} \ .
\end{aligned} \tag{19}
$$

Under the general coordinate transformation $\delta x^\mu = \epsilon^\mu$, $b^a_{\mu\nu}$ transform as

$$
\delta b^a_{\mu\nu} = \varphi^2(\partial_\mu\epsilon^\rho\eta^a_{\rho\nu} + \partial_\nu\epsilon^\rho\eta^a_{\mu\rho}) + \epsilon_\rho\partial_\rho b^a_{\mu\nu} + \partial_\mu\epsilon^\rho b^a_{\rho\nu} + \partial_\nu\epsilon^\rho b^a_{\mu\rho} \tag{20}
$$

since $\delta B^a_{\mu\nu} = \epsilon^\rho\partial_\rho B^a_{\mu\nu} + \partial_\mu\epsilon^\rho B^a_{\rho\nu} + \partial_\nu\epsilon^\rho B^a_{\mu\rho}$. Note that there appear inhomogeneous terms in Eq.(20). Therefore we can fix the local symmetry of the general coordinate transformation by choosing the following conditions:

$$
e = e^a = 0 \ . \tag{21}
$$

This gauge condition is a kind of temporal gauge since e and e^a are parts of b^a_{i4} (17). Then the action has the following form (the actions of the ghost fields are given in the next section.):

$$
\begin{aligned}
S \ \sim \ & \int d^4x[12g\varphi^2\tilde{a}^2 + 4g\varphi^2\tilde{a}^a\tilde{a}^a - 2g\varphi^2\tilde{a}^{ab}\tilde{a}^{ab} \\
& - \frac{3}{16g\varphi^2}(\partial_4\tilde{h})^2 - \frac{1}{4g\varphi^2}(\partial_4\tilde{h}^a)^2 + \frac{1}{8g\varphi^2}(\partial_4\tilde{h}^{ab})^2 \\
& - 8g\varphi^6(\partial_4^{-1}\tilde{\phi}^{ab})^2 \\
& + \frac{1}{32g\varphi^2}\{(\epsilon^{dcb}\partial_c e^{ad} + \epsilon^{dca}\partial_c e^{bd}) - 2\partial_4 e^{ab}\}^2] \ .
\end{aligned} \tag{22}
$$

After the partial integration, the last term can be decomposed orthogonally as follows:

$$
\begin{aligned}
& \int d^4x[\{(\epsilon^{dcb}\partial_c e^{ad} + \epsilon^{dca}\partial_c e^{bd}) - 2\partial_4 e^{ab}\}^2] \\
= & \int d^4x[e^{ab}\{-(2\triangle + 4\partial_4^2)I + A + B\}e^{cd}] \\
= & \int d^4x[e^{ab}\{-4\partial_4^2 M_1 - (\triangle + 4\partial_4^2)M_2 - 4\partial_4^2 M_3 \\
& -(4\triangle + 4\partial_4^2)M_4\}_{(ab)(cd)}e^{cd}] \qquad (23) \\
& (\triangle = \sum_{a=1}^{3}\partial_a\partial_a \ , \ \ M_i\, M_j = \delta_{ij}M_i)
\end{aligned}
$$

M_i's are given in Appendix A.

4 Measure and Ghost Actions

We now determine the measures of the fields, which preserves the $SU(2)$ gauge symmetry and the general covariance, by

$$
\begin{aligned}
(\delta B^a_{\mu\nu})^2 &= \int d^4x\epsilon^{\mu\nu\rho\sigma}\delta B^a_{\mu\nu}\delta B^a_{\rho\sigma} \\
(\delta A^a_\mu)^2 &= \int d^4x\epsilon^{\mu\nu\rho\sigma}\epsilon^{abc}B^a_{\mu\nu}\delta A^b_\rho\delta A^c_\sigma \\
(\delta\phi^{ab})^2 &= \int d^4x\epsilon^{\mu\nu\rho\sigma}B^a_{\mu\nu}B^a_{\rho\sigma}\delta\phi^{bc}\delta\phi^{bc}\ . \qquad (24)
\end{aligned}
$$

In the following, we impose the periodic boundary conditions for x^i with the periods L:

$$\int_{L^3} d^3x \equiv \int_{-\frac{L}{2}}^{+\frac{L}{2}} dx \int_{-\frac{L}{2}}^{+\frac{L}{2}} dy \int_{-\frac{L}{2}}^{+\frac{L}{2}} dz \qquad (25)$$

Since $B^a_{\mu\nu} \sim \varphi^2\eta^a_{\mu\nu}$ *i.e.* $e^A_\mu \sim \varphi\delta^A_\mu$ $(A = 1, \cdots, 4)$, we find $ds^2 \sim \varphi^2\sum_{\mu=1,\cdots,4} dx^\mu dx^\mu$. Therefore the volume of the universe is given by $(\varphi L)^3$, which can be determined if the effective potential of φ has non-trivial minimum. By defining new coordinates $\tilde{x}^\mu$ by $x^\mu = \varphi^{-1}\tilde{x}^\mu$, which gives $ds^2 \sim \sum_{\mu=1,\cdots,4} dx^\mu dx^\mu$, the measures in Eq.(24) can be rewritten by

$$(\delta B^a_{\mu\nu})^2 \ \sim \ \varphi^{-4}\int_{(\varphi L)^3} d^3\tilde{x}\int_0^{\frac{1}{kT}} d\tilde{x}^4\epsilon^{\mu\nu\rho\sigma}\delta B^a_{\mu\nu}\delta B^a_{\rho\sigma}$$

$$
\begin{aligned}
(\delta A^a_\mu)^2 &\sim \varphi^{-2}\int_{(\varphi L)^3} d^3\tilde{x}\int_0^{\frac{1}{kT}} d\tilde{x}^4 \epsilon^{\mu\nu\rho\sigma}\epsilon^{abc}\eta^a_{\mu\nu}\delta A^b_\rho \delta A^c_\sigma \\
(\delta\phi^{ab})^2 &\sim \int_{(\varphi L)^3} d^3\tilde{x}\int_0^{\frac{1}{kT}} d\tilde{x}^4 \epsilon^{\mu\nu\rho\sigma}\eta^a_{\mu\nu}\eta^a_{\rho\sigma}\delta\phi^{bc}\delta\phi^{bc}
\end{aligned} \tag{26}
$$

Here we have replaced $B^a_{\mu\nu}$ by $\varphi^2\eta^a_{\mu\nu}$ and neglected higher order terms with respect to the power of fields. We also impose the periodic boundary condition for $\tilde{x}^4$ with period $1/kT$ (k is the Boltzman constant.), *i.e.* we are now considering the field theory in the finite temperature T.

If we redefine the fields by

$$
\hat{B}^a_{\mu\nu} = \varphi^{-2}B^a_{\mu\nu} \ (\rightarrow e, h) \ , \tag{27}
$$

$$
\hat{A}^a_\mu = \varphi^{-1}A^a_\mu \ (\rightarrow a) \ , \tag{28}
$$

$$
\hat{\phi}^{ab} = \phi^{ab} \ , \tag{29}
$$

the φ dependence in the measures is absorbed and the action (22) has the following form

$$
\begin{aligned}
S = & \int_{(\varphi L)^3} d^3\tilde{x}\int_0^{\frac{1}{kT}} d\tilde{x}^4 \\
& \times\Big[12g\hat{a}^2 + 4g\hat{a}^a\hat{a}^a - 2g\hat{a}^{ab}\hat{a}^{ab} \\
& -\frac{3}{16g}(\tilde{\partial}_4\hat{h})^2 - \frac{1}{4g}(\tilde{\partial}_4\hat{h}^a)^2 + \frac{1}{8g}(\tilde{\partial}_4\hat{h}^{ab})^2 \\
& -8g(\tilde{\partial}_4^{-1}\hat{\phi}^{ab})^2 \\
& +\frac{1}{32g}\hat{e}^{ab}\{-4\tilde{\partial}_4^2 M_1 - (\tilde{\triangle} + 4\tilde{\partial}_4^2)M_2 - 4\tilde{\partial}_4^2 M_3 \\
& \qquad -(4\tilde{\triangle} + 4\tilde{\partial}_4^2)M_4\}_{(ab)(cd)}\hat{e}^{cd}\Big] \ .
\end{aligned} \tag{30}
$$

Note that there does not appear φ dependence in the Lagrangean density in the action (30).

We now consider the actions and measures for the ghost fields. By using a vector $k^\mu = (0, 0, 0, 1)$, the gauge fixing conditions (13) and (21) can be rewritten as follows,

$$
A^a_4 = k^\tau A^a_\tau = 0 \tag{31}
$$

$$
G^\mu = k^\tau\epsilon^{\mu\nu\rho\sigma}\eta^a_{\nu\rho}b^a_{\tau\sigma} \approx k^\tau\epsilon^{\mu\nu\rho\sigma}B^a_{\nu\rho}b^a_{\tau\sigma} = 0 \ . \tag{32}
$$

Since the inhomogeneous terms under the $SU(2)$ gauge transformation and the general coordinate transformation is given by

$$\begin{aligned} \delta A_4^a &= \partial_4 \theta^a + \cdots && (33) \\ \delta b_{\mu\nu}^a &= \varphi^2 (\eta_{\rho\nu}^a \partial_\mu \epsilon^\rho + \eta_{\mu\rho}^a \partial_\nu \epsilon^\rho) + \cdots \ , && (34) \end{aligned}$$

we find that the actions and measures of the (anti-)ghost fields (r^a, c^a), (p_α, q^α) are given by

$$\begin{aligned} S_{\rm ghost} &\sim \int d^4x \, \{ \epsilon^{\mu\nu\rho\sigma} B_{\mu\nu}^a B_{\rho\sigma}^a k^\tau r^b \partial_\tau c^b \\ &\quad + p_\mu k^\tau \epsilon^{\mu\nu\rho\sigma} B_{\mu\nu}^a (B_{\beta\sigma}^a \partial_\tau q^\beta + B_{\tau\beta}^a \partial_\sigma q^\beta) \} \\ &\sim \varphi^4 \int d^4x \{ \epsilon^{\mu\nu\rho\sigma} \eta_{\mu\nu}^a \eta_{\rho\sigma}^a k^\tau r^b \partial_\tau c^b \\ &\quad + p_\mu k^\tau \epsilon^{\mu\nu\rho\sigma} \eta_{\mu\nu}^a (\eta_{\beta\sigma}^a \partial_\tau q^\beta + \eta_{\tau\beta}^a \partial_\sigma q^\beta) \} && (35) \end{aligned}$$

$$\begin{aligned} M_{\rm ghost} &\sim \int d^4x \{ \epsilon^{\mu\nu\rho\sigma} B_{\mu\nu}^a B_{\rho\sigma}^a (\delta r^b \delta c^b + \delta p_\alpha \delta q^\alpha) \} \\ &\sim \varphi^4 \int d^4x (\delta r^a \delta c^a + \delta p_\alpha \delta q^\alpha) \ . && (36) \end{aligned}$$

Here we have kept only quadratic terms with respect to (anti-)ghost fields. By using the coordinate system $\{\tilde{x}^\mu\}$ and redefining the ghost fields (p_α, q^α) by $p_\alpha = \varphi \hat{p}_\alpha$ and $q^\alpha = \varphi^{-1} \hat{q}^\alpha$, the φ dependence of measures is absorbed and we obtain

$$\begin{aligned} S_{ghost} &\sim \varphi \int d^4\tilde{x} \{ \epsilon^{\mu\nu\rho\sigma} \eta_{\mu\nu}^a \eta_{\rho\sigma}^a k^\tau r^b \tilde{\partial}_\tau c^b \\ &\quad + \hat{p}_\mu k^\tau \epsilon^{\mu\nu\rho\sigma} \eta_{\mu\nu}^a (\eta_{\beta\sigma}^a \tilde{\partial}_\tau \hat{q}^\beta + \eta_{\tau\beta}^a \tilde{\partial}_\sigma \hat{q}^\beta) \} && (37) \\ M_{ghost} &\sim \int d^4\tilde{x} (\delta r^a \delta c^a + \delta \hat{p}_\alpha \delta \hat{q}^\alpha) \ . && (38) \end{aligned}$$

By redefining $\hat{q}_4 \to \hat{q}_4 - \frac{1}{3} \tilde{\partial}_4^{-1} \tilde{\partial}_a \, \hat{q}^a$, the second term in the action (37) is rewritten by

$$-2\varphi \int d^4\tilde{x} \{ \hat{p}_4 \, (\, 3 \tilde{\partial}_4 \hat{q}_4) \ + \ \hat{p}_i \, (\, \tilde{\partial}_4 \hat{q}^i + \epsilon^{ia\sigma} \tilde{\partial}_\sigma \hat{q}^a) \} \ . \qquad (39)$$

The one-loop contribution of ghost and anti-ghost fields (p^α, q_α) and (r^a, c^a) to the effective potential can be evaluated by using the following

formula:

$$
\begin{aligned}
\int d\beta d\gamma \mathrm{e}^{\int d^4\tilde{x}\beta D\gamma} &= \det D = \{\det D\}^{\frac{1}{2}}\{\det D\}^{\frac{1}{2}} \\
&= \{\det D\}^{\frac{1}{2}}\{\det{}^T D\}^{\frac{1}{2}} \\
&= \mathrm{e}^{\frac{1}{2}\,\mathrm{Tr}\,\ln D^{\,T} D} \\
&= \int d\beta d\bar{\beta} \mathrm{e}^{\int d^4\tilde{x}\beta D^{\,T} D\bar{\beta}} \qquad (40)
\end{aligned}
$$

Here (β, γ) represents (p^α, q_α) and (r^a, c^a). ($D = \varphi\tilde{\partial}_4$ for (p^α, q_α) and $D = D^a_b = \varphi(\tilde{\partial}_4\delta^a_b + \epsilon^{ab\sigma}\tilde{\partial}_\sigma)$ for (r^a, c^a).) The explicit form of $D^{\,T}D$ for (r^a, c^a), which can be orthogonally decomposed, is given in Appendix B.

5 Effective Potential

The one-loop contributions of any fields to the effective potential take the form of $\mathrm{Tr}\ln\{A\triangle + B\partial_4^2\}$, which is quartically divergent. We evaluate this quantity by using the following regularization:

$$
\begin{aligned}
& F(a, b; \alpha, \beta; \epsilon) \\
= & \sum_{\substack{n_1,n_2,n_3,m=-\infty \\ (n_1,n_2,n_3,m\neq 0)}}^{+\infty} \ln\{a(n_1^2+n_2^2+n_3^2)+bm^2\}e^{-\epsilon\{\alpha(n_1^2+n_2^2+n_3^2)+\beta m^2\}} \ . \qquad (41)
\end{aligned}
$$

Here ϵ is a cut-off parameter for the regularization and the c-number coefficients α and β are chosen so as to keep the invariance of the local symmetry . By using the following formulae,

$$
\ln Q = -\frac{\partial}{\partial\xi}\frac{1}{\Gamma(\xi)}\int_0^\infty ds s^{\xi-1}e^{-Qs}\Big|_{\xi=0} \qquad (42)
$$

$$
\vartheta_3(\upsilon, \tau) = \sum_{n=-\infty}^{\infty}(e^{\tau\pi i})^{n^2}(e^{\upsilon\pi i})^{2n} \ , \qquad (43)
$$

Eq.(41) is rewritten by

$$
\begin{aligned}
F(a, b; \alpha, \beta; \epsilon) &= -\frac{\partial}{\partial\xi}\Big[\frac{\epsilon^\xi}{\Gamma(\xi)}\int_0^\infty dt\, t^{\xi-1} \qquad (44) \\
&\times\Big\{\vartheta_3(0, \frac{i}{\pi}\epsilon(at+\alpha))^3\vartheta_3(0, \frac{i}{\pi}\epsilon(bt+\beta)) - 1\Big\}\Big]\Big|_{\xi=0} \ .
\end{aligned}
$$

Since the theta function $\vartheta_3(0, \frac{ix}{\pi})$ has the following properties,

$$\left\{ \begin{array}{ccc} \lim_{x\to\infty} \vartheta_3(0, \frac{ix}{\pi}) & = & 1 \\ \vartheta_3(0, \frac{ix}{\pi}) & = & (\frac{\pi}{x})^{1/2} \vartheta_3(0, \frac{\pi}{x} i) \\ & \stackrel{x\to 0}{\longrightarrow} & (\frac{\pi}{x})^{1/2} \quad + o(x^n) \end{array} \right. , \tag{45}$$

we can use the following approximation if we are interested in the leading term with respect to the cut-off parameter ϵ:

$$F(a, b; \alpha, \beta; \epsilon) \sim -\frac{\partial}{\partial \xi} \left[\frac{\epsilon^\xi}{\Gamma(\xi)} \int_0^\infty dt\, t^{\xi-1} \frac{\pi^2}{\epsilon^2 (at+\alpha)^{3/2} (bt+\beta)^{1/2}} \right] \Big|_{\xi=0} . \tag{46}$$

By changing the variable $t = \alpha s/a$, we obtain

$$\begin{aligned} F(a, b; \alpha, \beta; \epsilon) \quad \sim \quad & -\frac{\partial}{\partial \xi} \Big[\frac{\epsilon^{\xi-2} \pi^2}{\Gamma(\xi) a^{3/2} b^{1/2}} \left(\frac{\alpha}{a} \right)^{\xi-2} \\ & \times \int_0^\infty ds\, \frac{s^{\xi-1}}{(s+1)^{3/2} (s + a\beta/b\alpha)^{1/2}} \Big] \Big|_{\xi=0} . \end{aligned} \tag{47}$$

The integration

$$f(x, \xi) \equiv \int_0^\infty ds\, s^{\xi-1} \frac{1}{(s+1)^{3/2} (s+x)^{1/2}} \tag{48}$$

is given by Gauss' hypergeometric function:

$$f(x, \xi) = \frac{\pi(1-\xi)}{\sin(\pi\xi)} \mathcal{F}(2-\xi, \frac{1}{2}, 2; 1-x) . \tag{49}$$

By expanding $f(x, \xi)$ with respect to ξ, we find

$$\begin{aligned} f(x) \quad = \quad & \left[\frac{1}{\xi} s^\xi \frac{1}{(s+1)^{3/2} (s+x)^{1/2}} \right]_{s=0}^\infty \Big|_{\xi=0} \\ & -\frac{1}{\xi} \int_0^\infty s^\xi \left\{ -\frac{3}{2} \frac{1}{(s+1)^{5/2} (s+x)^{1/2}} - \frac{1}{2} \frac{1}{(s+1)^{3/2} (s+x)^{3/2}} \right\} \Big|_{\xi=0} \\ \approx \quad & -\frac{1}{\xi} \int_0^\infty ds\, (1 + \xi \ln s + O(\xi^n)) \{ \quad \} \Big|_{\xi=0} \\ \equiv \quad & f_{-1}(x) \xi^{-1} + f_0(x) \xi^0 + f_1(x) \xi + \cdots \end{aligned} \tag{50}$$

f_{-1} is given by the incomplete beta function $B_z(p,q)$. (Explicit form for f_{-1} is given in Appendix C. By using Eq.(50), we find the following expression of $F(a,b;\alpha,\beta;\epsilon)$:

$$
\begin{aligned}
F(a,b;\alpha,\beta;\epsilon) &\sim -\frac{\partial}{\partial\xi}\left[\frac{(1+\xi\ln\epsilon+\mathcal{O}(\xi^n))\pi^2}{(\frac{1}{\xi}-\gamma+\mathcal{O}(\xi^n))a^{3/2}b^{1/2}}\left(\frac{\alpha}{a}\right)^{\xi-2} f(\frac{a\beta}{b\alpha})\right]\Big|_{\xi=0} \\
&= \frac{-\pi^2}{\epsilon^2 a^{3/2}b^{1/2}}\left(\frac{\alpha}{a}\right)^{-2} \\
&\quad \times\left\{f_{-1}(\frac{a\beta}{b\alpha})(\ln\frac{\alpha}{a}+\ln\epsilon+\gamma)+f_0(\frac{a\beta}{b\alpha})\right\} \ . \qquad (51)
\end{aligned}
$$

In order to determine the coefficients α and β, which are the parameters for the regularization, we consider actions invariant under the general coordinate transformation. In case of the anti-ghost field r^a, the invariant action is given by,

$$
\int d^4x\sqrt{-g}g^{\mu\nu}\partial_\mu\bar{r}^a\partial_\nu r^a \sim \int d^4\tilde{x}\eta^{\mu\nu}\tilde{\partial}_\mu\bar{r}^a\tilde{\partial}_\nu r^a \ . \qquad (52)
$$

Here $\bar{r}^a$ is an anti-self-dual partner of r^a in Eq.(40). Since the eigenvalues of $\tilde{\triangle}$ and $\tilde{\partial}_4^2$ are given by

$$
\begin{aligned}
\tilde{\triangle}^2 &= -\frac{(2\pi)^2}{L^2\varphi^2}(n_1^2+n_2^2+n_3^2) \qquad (53) \\
\tilde{\partial}_4^2 &= -(2\pi kT)^2m^2 \ , \qquad (54)
\end{aligned}
$$

we find α and β corresponding to r^a,

$$
\alpha=\frac{(2\pi)^2}{L^2\varphi^2} \ , \ \beta=(2\pi kT)^2 \ . \qquad (55)
$$

In a similar way, we find the actions of $(p_\alpha \ , \ \bar{p}_\alpha)$, $B^a_{\alpha\beta}$ and ϕ^{ab}

$$
\begin{aligned}
\int d^4x\sqrt{-g}g^{\mu\nu}g^{\rho\sigma}D_\mu p_\rho D_\nu\bar{p}_\sigma &\sim \int d^4\tilde{x}\epsilon^{\mu\nu\rho\sigma}\tilde{\partial}_\mu\hat{p}_\rho\tilde{\partial}_\nu\hat{\bar{p}}_\sigma \qquad (56) \\
\int d^4xg^{\mu\nu}\epsilon^{\alpha\beta\gamma\delta}\partial_\mu b^a_{\alpha\beta}\partial_\nu b^a_{\gamma\delta} &\sim \int d^4\tilde{x}\eta^{\mu\nu}\epsilon^{\alpha\beta\gamma\delta}\tilde{\partial}_\mu\hat{b}^a_{\alpha\beta}\tilde{\partial}_\nu\hat{b}^a_{\gamma\delta} \qquad (57) \\
\int d^4x\sqrt{-g}g^{\mu\nu}\partial_\mu\phi^{ab}\partial_\nu\phi^{ab} &\sim \int d^4\tilde{x}\eta^{\mu\nu}\tilde{\partial}_\mu\phi^{ab}\tilde{\partial}_\nu\phi^{ab} \qquad (58) \\
\int d^4x\epsilon^{\alpha\beta\gamma\delta}\partial_\alpha A^a_\beta\partial_\gamma A^a_\delta &\sim \int d^4\tilde{x}\epsilon^{\alpha\beta\gamma\delta}\tilde{\partial}_\alpha\hat{A}^a_\beta\tilde{\partial}_\gamma\hat{A}^a_\delta \qquad (59)
\end{aligned}
$$

and the coefficients α and β

$$\alpha = \frac{(2\pi)^2}{L^2\varphi^2} \ , \qquad \beta = (2\pi kT)^2 \ . \tag{60}$$

By considering the contributions from all the fields, we find that the effective potential has the following form:

$$V(\varphi, T) \sim \frac{(L\varphi)^3}{\epsilon^2 kT} \left\{ C_1 \ln(\mu^2\epsilon) + C_2 \ln \varphi^2 + c\text{−number} \right\} \ . \tag{61}$$

Here μ is a renormalization point. Note that only ghosts can contribute to the coefficients C_2. If $C_2 = 0$, the quartically divergent part of the effective potential can be removed by the renormalization of the cosmological constant and the potential would not have any physical meaning. $C_2 \neq 0$ means that there is a divergence which cannot be renormalized and there is a kind of anomaly. If the cut-off scale has any physical meaning, *e.g.* Planck scale coming from string theory, the effective potential would give the following physical implication: The effective potential has only one non-trivial ($\varphi \neq 0$) minimum. In the low temperature ($T \to 0$), the minimum is deep and the metric does not fluctuate. This means that the metric has a non-trivial classical value. On the other hand, in the high temperature, the effective potential is flat and the fluctuation of the space-time metric is large. Therefore this effective potential might explain why there is the universe at present.

6 Summary

In the framework of two-form gravity, which is classically equivalent to Einstein gravity, we have calculated the one-loop effective potential for the conformal factor of metric in the finite volume and in the finite temperature by choosing a temporal gauge fixing condition. There appears a quartically divergent term which cannot be removed by the renormalization of the cosmological term and we have found a non-trivial minimum in the effective potential. If the cut-off scale has any physical meaning, *e.g.* the Planck scale coming from string theory, this minimum might explain why the space-time is generated, *i.e.* why the classical metric has a non-trivial value.

The two-form gravity theory might be an low energy effective theory of string theory since the Kalb-Ramond symmetry, which is characteristic to the two-form gravity, is stringy symmetry [4].

Appendix A. Orthogonal Decomposition for the Kinetic Terms of e^{ab}

By partially integrating

$$\{\epsilon^{bdc}\partial_c\hat{e}^{ad} + \epsilon^{adc}\partial_c\hat{e}^{bd} + 2\partial_4\hat{e}^{ab}\}^2 \ , \tag{62}$$

we obtain

$$\hat{e}^{ab}\{-(2\triangle + 4\partial_4^2)I_{(ab)(cd)} + A_{(ab)(cd)} + B_{(ab)(cd)}\}\hat{e}^{cd} \ . \tag{63}$$

Here I, A and B are defined by,

$$\begin{cases} I_{(ab)(cd)} & \equiv & \frac{1}{2}(\delta^{ac}\delta^{bd} + \delta^{ad}\delta^{bc}) - \frac{1}{3}\delta^{ab}\delta^{cd} \\ A_{(ab)(cd)} & \equiv & \frac{1}{2}(\delta^{ac}\partial_b\partial_d + \delta^{ad}\partial_b\partial_c + \delta^{bc}\partial_a\partial_d + \delta^{bd}\partial_a\partial_c) \\ & & -\frac{2}{3}(\delta^{cd}\partial_a\partial_b + \delta^{ab}\partial_c\partial_d) + \frac{2}{9}\delta^{ab}\delta^{cd}\triangle \\ B_{(ab)(cd)} & \equiv & \epsilon^{bce}\epsilon^{adf}\partial_e\partial_f + \epsilon^{ace}\epsilon^{bdf}\partial_e\partial_f \\ & & +\frac{2}{3}(\delta^{cd}\partial_a\partial_b + \delta^{ab}\partial_c\partial_d) - \frac{8}{9}\delta^{ab}\delta^{cd}\triangle \end{cases} \ . \tag{64}$$

If we define

$$C_{(ab)(cd)} \equiv \partial_a\partial_b\partial_c\partial_d - \frac{1}{3}(\partial_a\partial_b\delta^{cd}\triangle + \delta^{ab}\partial_c\partial_d\triangle) + \frac{1}{9}\delta^{ab}\delta^{cd}\triangle^2 \ , \tag{65}$$

I, A, B and C satisfy the following algebra:

$$\begin{cases} A^2 = \triangle A + \frac{2}{3}C & AB = BA = \frac{4}{3}C \\ B^2 = 4\triangle I - 4\triangle A + \frac{8}{3}C & BC = CB = \frac{2}{3}\triangle C \\ C^2 = \frac{2}{3}\triangle^2 C & CA = AC = \frac{4}{3}\triangle C \end{cases} \tag{66}$$

$$\mathrm{tr}\, I = I_{(ab)(cd)} = 5 \ , \ \ \mathrm{tr}\, A = \frac{10}{3}\triangle \ , \ \ \mathrm{tr}\, B = -\frac{10}{3}\triangle \ , \ \ \mathrm{tr}\, C = \frac{2}{3}\triangle^2 \ . \tag{67}$$

If we define M_i's by

$$\begin{aligned} & M_1 \equiv \frac{3}{2\triangle^2}C \ , \ \ M_2 \equiv \frac{1}{\triangle}(A - \frac{2}{\triangle}C) \ , \ \ M_3 \equiv \frac{1}{\triangle}(B - 2A + 2\triangle I) \ , \\ & M_4 \equiv -\frac{1}{4\triangle}(B + 2A - 2\triangle I - \frac{2}{\triangle}C) \ , \end{aligned} \tag{68}$$

M_i's satify the equation $M_iM_j = \delta^{ij}M_i$ and we find

$$f(\sum_{i=1}^{4}\alpha_iM^i) = \sum_{i=1}^{4}f(\alpha_i)M^i \tag{69}$$

for arbitrary function $f(x)$. By using M, we can express I, A, B and C by

$$\begin{cases} I &= M_1 + M_2 + M_3 + M_4 \\ A &= \triangle(M_2 + \frac{4}{3}M_1) \\ B &= 2\triangle(M_3 - M_4 + \frac{1}{3}M_1) \\ C &= \frac{2}{3}\triangle^2 M_1 \end{cases} , \tag{70}$$

and we obtain the following orthogonal decomposition

$$\begin{aligned} M &= -2(\triangle + 4\partial_4^2)I + A + B \\ &= -4\partial_4^2 M_1 - (\triangle + 4\partial_4^2)M_2 - \partial_4^2 M_3 - (4\triangle + 4\partial_4^2)M_4 \end{aligned} \tag{71}$$

$$\mathrm{tr}\, M_1 = 1\ ,\ \ \mathrm{tr}\, M_2 = 2\ ,\ \ \mathrm{tr}\, M_3 = 0\ ,\ \ \mathrm{tr}\, M_4 = 2\ . \tag{72}$$

Appendix B. Orthogonal Decomposition for the Kinetic Terms of r^a

In order to evaluate ($D_b^a \equiv \tilde{\partial}_4 \delta_b^a + \epsilon^{abc}\tilde{\partial}_c$),

$$\begin{aligned} \int dpdq\, \mathrm{e}^{\{p^a D_b^a q^b\}} &= \det D_b^a = \{\det D\}^{\frac{1}{2}}\{\det D\}^{\frac{1}{2}} \\ &= \{\det\ D\}^{\frac{1}{2}}\{\det{}^T D\}^{\frac{1}{2}} \\ &= \mathrm{e}^{\frac{1}{2}\,\mathrm{Tr}\,\ln\, D\,^T D}\ , \end{aligned} \tag{73}$$

we orthogonalize $D\,^T D$, which is explicitly given by

$$\begin{aligned} D_b^a\,{}^T D_c^b &= -\partial_4^2 \delta_c^a + \epsilon^{acd}\tilde{\partial}_4\tilde{\partial}_d - \epsilon^{acd}\tilde{\partial}_4\tilde{\partial}_d + \epsilon^{abd}\epsilon^{bce}\tilde{\partial}_d\tilde{\partial}_e \\ &= -\tilde{\partial}_4\delta_c^a + \tilde{\partial}_c\tilde{\partial}_a - \delta^{ac}\tilde{\triangle}\ . \end{aligned} \tag{74}$$

If we define

$$N_{(ab)}^{(1)} = \frac{1}{\tilde{\triangle}}\tilde{\partial}_a\tilde{\partial}_b\ ,\ \ N_{(ab)}^{(2)} = \delta_{ab} - \frac{1}{\tilde{\triangle}}\tilde{\partial}_a\tilde{\partial}_b\ , \tag{75}$$

$N^{(i)}$'s satisfy the following relation:

$$N^{(1)}N^{(2)} = 0\ ,\ \ N^{(1)}N^{(1)} = N^{(1)}\ ,\ \ N^{(2)}N^{(2)} = N^{(2)} \tag{76}$$

and we find

$$D_b^{aT} D_c^b = -\tilde{\partial}_4^2 N_{(ac)}^{(1)} - (\tilde{\triangle} + \tilde{\partial}_4^2)N_{(ac)}^{(2)}\ . \tag{77}$$

Appendix C. The Evaluation of the Function $f(x, \xi)$

The incomplete beta function is defined by

$$B_z(p,q) \equiv \int_0^z dt\, t^{p-1}(1-t)^{q-1} \ . \tag{78}$$

By changing the variable $t = s/(s+1)$, we obtain

$$B_z(p,q) = \int_0^{\frac{z}{1-z}} ds = B(p,q) - \int_{\frac{z}{1-z}}^{\infty} ds\, \frac{s^{p-1}}{(1+s)^{p+q}} \ . \tag{79}$$

The change of the variable $s \to z(1+s)/(1-z)$ gives

$$\begin{aligned} B_z(p,q) &= B(p,q) - \int_0^\infty ds \frac{(s+\frac{z}{1-z})^{p-1}}{(s+\frac{1}{1-z})^{p+q}} \\ &= B(p,q) - \int_0^\infty ds (1-z)^q \frac{1}{(s+z)^{1-p}(s+1)^{p+q}} \end{aligned} \tag{80}$$

and we find

$$\int_0^\infty \frac{1}{(s+z)^{1-p}(s+1)^{p+q}} = \frac{1}{(1-z)^q} \left(B(p,q) - B_z(p,q)\right) \tag{81}$$

and

$$\begin{aligned} f_{-1} &= \frac{1}{(1-x)^2} \Big\{ -\frac{3}{2} \left(B(\frac{1}{2},2) - B_z(\frac{1}{2},2) \right) \\ &\quad -\frac{1}{2} \left(B(-\frac{1}{2},2) - B_z(-\frac{1}{2},2) \right) \Big\} \ . \end{aligned} \tag{82}$$

The function $f(x,\xi)$, which is defined by

$$\begin{aligned} f(x,\xi) &\equiv \int_0^\infty ds\, s^{\xi-1} \frac{1}{(s+1)^{3/2}(s+x)^{1/2}} \\ &= \int_0^\infty ds\, s^{\xi-1}(s+1)^{-2} \left(1 + \frac{x-1}{s+1}\right)^{-1/2} \end{aligned} \tag{83}$$

can be expressed by Gauss' hypergeometic function. By using the binomial expansion and the definition of the beta function

$$(1+x)^\alpha = \sum_{n=0}^{\infty} \frac{\Gamma(\alpha+1)}{\Gamma(\alpha-n+1)\Gamma(n+1)} x^n \tag{84}$$

$$B(p,q) = \frac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)} = \int_0^\infty dt \frac{t^{p-1}}{(1+t)^{p+q}} \ , \tag{85}$$

we obtain the following expression

$$\begin{aligned} f(x) &= \sum_{n=0}^{\infty} \int_0^\infty ds\, s^{\xi-1}\,(s+1)^{-n-2} \frac{\Gamma(\frac{1}{2})}{\Gamma(\frac{1}{2})\Gamma(n+1)}(x-1)^n \\ &= \sum_{n=0}^{\infty} \frac{\Gamma(\xi)\Gamma(-\xi+n+2)}{\Gamma(n+2)} \frac{\Gamma(\frac{1}{2})}{\Gamma(\frac{1}{2}-n)\Gamma(n+1)}(x-1)^n \\ &= \frac{\Gamma(\xi)\Gamma(\frac{1}{2})}{\sqrt{\pi}} \sum_{n=0}^{\infty} \frac{\Gamma(2-\xi+n)\Gamma(n+\frac{1}{2})}{\Gamma(n+2)} \frac{(1-x)^n}{n!} \ . \end{aligned} \tag{86}$$

Here we have used a formula $\Gamma(z+\frac{1}{2})\Gamma(\frac{1}{2}-z) = \frac{\pi}{\cos \pi z}$. By using the definition of Gauss' hypergeometric function

$$\mathcal{F}(\alpha,\beta,\gamma;z) = \frac{\Gamma(\gamma)}{\Gamma(\alpha)\Gamma(\beta)} \sum_{n=0}^{\infty} \frac{\Gamma(\alpha+n)\Gamma(\beta+n)}{\Gamma(\gamma+n)} \frac{z^n}{n} \ , \tag{87}$$

we obtain

$$\begin{aligned} f(x) &= \frac{\Gamma(\xi)\Gamma(\frac{1}{2})}{\pi} \frac{\Gamma(2-\xi)\Gamma(\frac{1}{2})}{\Gamma(2)} \mathcal{F}(2-\xi,\ \frac{1}{2}\ ,\ 2\,;1-x) \\ &= \frac{\pi(1-\xi)}{\sin(\pi\xi)} \mathcal{F}(2-\xi,\ \frac{1}{2}\ ,\ 2\,;1-x) \ . \end{aligned} \tag{88}$$

Since $\mathcal{F}(2,\frac{1}{2},2;1-x) = x^{-1/2}$, we obtain the following Taylor expansion:

$$f(x,\xi) = \frac{x^{-1/2}}{\xi} + \left\{ -x^{-1/2} + \partial_\xi \mathcal{F}(2-\xi,\ \frac{1}{2},\ 2\,;1-x)\Big|_{\xi=0} \right\} \tag{89}$$

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